

Universal Short-Range Repulsion in the Baryon System Originating from the Confinement

— Approach in String-Junction Model ^{*)} —

Ryozo TAMAGAKI^{**)}

Kamitakano Maeda-Cho 26-5, Kyoto 606-0097, Japan

Aim of this study is to show a way to unifiedly understand the origin of repulsive core of baryon-baryon interaction and the origin of universal repulsion of three-baryon interaction whose importance has recently become noticeable. “Universal” means its effect independent of flavor and spin of baryons. As shown in recent lattice QCD calculations, the confinement mechanism in the single baryon is realized in the Y-shaped structure of the strings embodying the squeezed color flux-tubes that consists of a junction at the central position and three oriented strings linking the quarks with the junction. Here we adopt a string-junction model suitable for description of such baryonic structure, in order to find out substantial features that the confinement mechanism persists in multi-baryon systems when baryons closely approach to each other. First we clarify an originating mechanism of the repulsive core of baryon-baryon interaction, and next by extending this approach to the three-baryon system, we derive a universal short-range three-body repulsion. Key concept in the study lies in the recognition that such short-range repulsion appears as latent effect implying the energy necessary for full overlap of baryons, that is, the energy for creation of the junction and anti-junction pair which leads to specific exotic baryon states of gluonic excitation. The short-range interaction derived in such a way is universal, because the effect comes from the color degrees of freedom. This three-body repulsion prevents the equation of state from the dramatic softening due to hyperon-mixing in neutron-star matter, and thus we can dissolve the difficulty that otherwise theory contradicts with the observations on the masses of neutron stars.

§1. Introduction

In recent studies of neutron star (NS) matter with a hyperon-mixed core at high density, Takatsuka et al.²⁾ have shown that universal repulsion of three-baryon interaction (3BI) is indispensable to avoid the dramatic softening of equation of state (EOS) due to the hyperon-mixing, which leads to the contradiction to the observations on the masses of NSs.^{***)} Here “universal” means that the repulsion is independent of flavor and spin. This recognition is significant, although phenomenological, because the statement holds almost independently of the models describing NS matter. Importance of the recognition is similar to the case of the repulsive core model for nuclear force proposed by Jastrow in 1951,³⁾ who succeeded in explaining the high energy nucleon-nucleon scattering data at that time without violation of the charge-independence of nuclear force. Therefore it is a very interesting problem to study the origin of such a universal 3BI repulsion. In three-body force, there is

^{*)} Preliminarily reported in Ref. 1).

^{**)} E-mail: tama-ktn@nike.eonet.ne.jp

^{***)} We use the following abbreviations; NS for neutron star, 3BI for three-baryon interaction, EOS for equation of state, and SJM for string-junction model.

3BI of meson-exchange nature acting in the intermediate region as well as 3BI acting at small inter-baryon distances. In this paper we concentrate on 3BI of short-range nature.

Recently we have studied the problem whether or not the 3BI due to the two-pion exchange via Δ -excitation gives rise a universally repulsive effect in high-density neutron star matter, and obtained the following results.⁴⁾ Although this 3BI acts repulsively in pure neutron matter, it does not provide a repulsive effect at high density strong enough to prevent the EOS of NSs from the dramatic softening due to the hyperon-mixing, because it does not acts on the admixed Λ -particle due to its spin-isospin dependence. Even though there may be other possibilities of mesonic origin, to get universally repulsive effects in 3BI is quite open.

Noticing the property that the repulsive 3BI needed phenomenologically is common for all the baryons, we study the problem from the viewpoint of the quark confinement, the nonperturbative effect inherent in the color degrees of freedom in QCD. In treating the baryon system, we should adopt a model which well describes the confinement mechanism in the single baryon. In recent studies with use of the lattice QCD calculations, Suganuma et al.⁵⁾ have shown that the confinement mechanism in the single baryon is represented as the Y-shaped structure consisting of a junction at the central point and three oriented strings embodying the squeezed color flux-tubes. The junction plays a role to neutralize the color and the strings stem from the junction and end at the quarks. Such a Y-shaped string-junction picture of the single baryon was firstly pointed out by Nambu in 1973,⁶⁾ and adopted as one of key structural elements in the string-junction model developed in the 1970's.⁷⁾ The recent lattice QCD calculations have given the basis that this picture for the confinement in the single baryon is real. In the present paper we study the problem by adopting the string-junction model we abbreviate as SJM.

At first, developing a previous study based on the SJM by the author in 1982,⁸⁾ we show a way to understand the originating mechanism of the repulsive core in the two-baryon system, where we assume the existence of exotic dibaryon states predicted by the SJM. Extending such a line of approach to three baryon system, we derive the universal repulsion of 3BI with short-range nature in the SJM. In the next step we study the effects from this part of 3BI on the EOS of neutron matter, by using the framework constructing an effective density-dependent two-body potential from 3BI. Because the resulting 3BI is universal, its effect in the NS matter does not change with the hyperon-mixing. Finally we show that the EOS obtained by including the effects from this universal repulsion of 3BI is similar to the sufficiently stiff EOS given by Takatsuka et al.⁹⁾ Thus we can avoid the dramatic softening of EOS due to the hyperon-mixing which otherwise leads to the contradiction with the observations on NS masses.

This paper is organized as follows. After a brief description on the string-junction model in §2, we show the way to understand the origin of the BB repulsive core in §3. In §4, the derivation of the universal repulsion of 3BI in the SJM is described, the density-dependent effective two-body potential is derived from this 3BI repulsion and results of numerical calculations are shown. Then in §5, the effects on the EOS of NS matter are numerically calculated and the results are shown. Finally several

related points are discussed in §6. Concluding remarks are given in the last section.

§2. Brief description of the string-junction model (SJM)

In this section, we briefly explain the SJM and give mass formula of hadrons which are used in later sections.

2.1. Description of ordinary hadrons in the SJM

The SJM was developed in the later half of the 1970's, following the studies of hadron reactions with use of quark lines and their rearrangement diagrams.^{7),10)} A series of studies on the SJM by Imachi, Otsuki and Toyoda^{11)–13)} are to be especially noted. Study of the present paper is performed on the basis of these works. In the SJM the color flux-tubes embodying the confinement mechanism are illustrated by strings with orientation, as in Fig. 1. For mesons, we define the orientation of strings as stemming from the anti-quark (\bar{q}) and ending at the quark (q). (The reverse choice is equally possible, as this is matter of definition.) The confinement mechanism is represented by formation of a closed net of oriented string(s). An ordinary meson is drawn by a single string as shown for M in Fig. 1. An ordinary baryon (anti-baryon) is drawn by the Y-shaped net with three strings stemming from a junction J (ending at an anti-junction \bar{J}), as shown for B (\bar{B}) in Fig. 1. The arrow attached to the string represents its orientation that means the predisposition of the order in appearance of q (denoted by \circ) and \bar{q} (denoted by \bullet) when we cut the string. This holds also for the case that q and/or \bar{q} do not appear at the end. Following our definition, q appears ahead of the arrow and \bar{q} appears behind the arrow. We simply write the ordinary meson of $(q\bar{q})$ as M , the ordinary baryon of $(qqqJ)$ as B and the ordinary anti-baryon of $(\bar{q}\bar{q}\bar{q}\bar{J})$ as \bar{B} , where (\cdots) denotes the color singlet object.

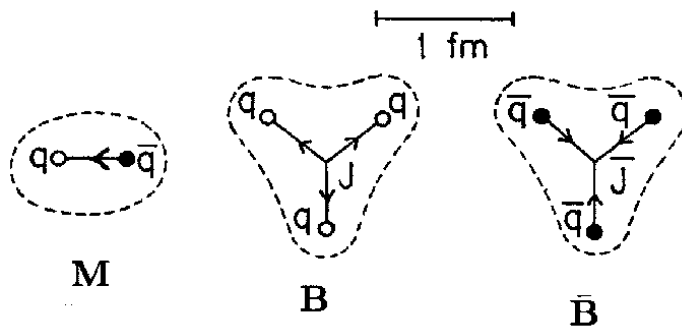


Fig. 1. Pictorial view of the ordinary hadrons; M , B and \bar{B} . The figures do not represent individual states of hadrons, but rather imply the states from which actual states are generated as Regge trajectories after the angular momentum projection, like the so-called intrinsic states in deformed nuclei. When we superpose various rotated configurations in the body-fixed frame, there appear the states spreading smoothly. The dotted curves illustrate the region supposed in the bag model.

The string structure for M has been noticed as a suitable representation for the color flux-tube since the early stage of QCD study. For the structure of B , Suganuma

and others have shown, in their recent lattice QCD calculations, that the junction structure of the Y-shaped strings is real.^{5),14)} This recognition has given the sound basis to such a view adopted in the SJM studies in the 1970's. The results of these QCD simulations are summarized in the energy expression of a potential form:⁵⁾

$$V_{q\bar{q}}(r) = \sigma_{q\bar{q}} r - \frac{A_{q\bar{q}}}{r} + C_{q\bar{q}}, \quad (2.1)$$

$$V_{3q} = \sigma_{3q} L_{\min} - A_{3q} \sum_{i < j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} + C_{3q}, \quad (2.2)$$

$$\sigma_{3q} \simeq 0.89 \text{ GeV/fm} \simeq \sigma_{q\bar{q}}, \quad A_{3q} \simeq \frac{1}{2} A_{q\bar{q}}. \quad (2.3)$$

Here r for M is the distance between q and \bar{q} , and L_{\min} for B is the minimum of the total length of the color flux-tubes linking three quarks with J . The 1st term of V_{3q} is the energy of the strings with the string tension almost equal to that of the q - \bar{q} case; this means the universality of the string tension. The 2nd term represents the color Coulomb-type attraction of the one-gluon exchange nature, as inferred from the relation $A_{3q} \simeq \frac{1}{2} A_{q\bar{q}}$. Such presentation has been shown to be valid over the actual range of $L_{\min} \sim (0.5 - 1.5) \text{ fm}$. The lattice QCD calculations have shown the profile of this Y-shaped string-junction in the action density,¹⁵⁾ and the actual length of the strings from the junction to the quarks, $\ell_s \simeq 0.5 \text{ fm}$,¹⁶⁾ that is, $L_{\min} \simeq 1.5 \text{ fm}$ for the equilateral configuration of the quarks.

2.2. Mass formula for hadrons in the SJM

Usually the hadrons other than M , B and \bar{B} are called exotic hadrons. This terminology is somewhat ambiguous, because there possibly exist many kinds of hadronic states, which are not simply described in the form of $(q\bar{q})$, $(qqqJ)$ and $(\bar{q}\bar{q}\bar{q}\bar{J})$ but not necessarily exotic, e.g., molecular states of the ordinary hadrons. In the SJM, we can discriminate exotic hadrons from nonexotic hadrons; exotic hadrons are characterized by the appearance of the interjunction string (abbreviated as IJ) which connects J and \bar{J} . Hadrons are constructed by three kinds of building blocks, that is, the quark q (anti-quark \bar{q}) with one oriented string, the junction J (anti-junction \bar{J}) with three strings and the string of the inter-junction IJ . We define the total number of quarks (the sum of the numbers of q and \bar{q}) denoted as N_q , the total number of junctions (the sum of the numbers of J and \bar{J}) by N_J and the total number of interjunctions by N_{IJ} . Then we denote the energy of each building block by m_q , m_J and m_{IJ} , respectively, where we take the same mass for q and \bar{q} and the same energy for J and \bar{J} .

The mass formula in the SJM are given in the simplest form, as follows:¹³⁾

$$m(N_q, N_J, N_{IJ}) = m_q N_q + m_J N_J + m_{IJ} N_{IJ} = m_B N_J - \delta N_{IJ}, \quad (2.4)$$

where we use the relations,

$$N_q = 3N_J - 2N_{IJ} \quad (N_J \neq 0), \quad m_B \equiv 3m_q + m_J, \quad \delta \equiv 2m_q - m_{IJ}. \quad (2.5)$$

The mass m_q represents the sum of the current quark mass and the energy of an attached string. For the light quarks (u , d and s) the string energy is the main part of m_q . In this paper we treat such a case. The mass m_B represents the average of the ground state of the single baryon. If $|\delta|$ is much smaller than the baryon mass $m_B \sim 1$ GeV, the energies of hadrons are mainly given by $m_B N_J$ and spread by the secondary contribution $-\delta N_{IJ}$. Estimate of magnitude of δ is discussed later. For the time being, we proceed under the assumption of the property $|\delta| \ll m_B$, which is proved in §6.

The most important point in this mass formula lies in the property that the order of a hadron mass is determined by the number of the junctions as $m(N_q, N_J, N_{IJ}) \sim N_J$ GeV, which we preferentially use in the following arguments.

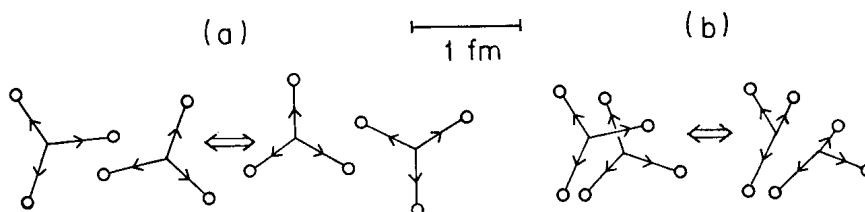


Fig. 2. Possible features of strings in approaching two baryons; (a) two adjacent strings change their connections with the nearest junctions to make the total length of strings minimum (flip-flop process) and (b) at closer approach, two B suffer strong deformation but the strings still keep the Y-shaped structure.@@@

2.3. Excitation of ordinary hadrons

A typical order of magnitude for increase of energy of M and B is considered as that of the masses of vector mesons such as ρ and ω . Meson production as fission of the string takes place for the elongation of a string with the order of $0.5 - 0.7$ fm in M and B . Excitation with such order of magnitude is also possible by deformation of the flux-tube and by swing in a relative angle between strings, which are regarded as vibrational modes in a broad sense. These are the excitations keeping the basic structure of M and B , and the substantial features of the confinement do not change.

At close approach of two baryons, when two Y-shaped strings begin to overlap, the rearrangement between two adjacent strings takes place to make deviation of the total length of the strings minimum, as shown in Fig. 2(a), which is called the “flip-flop” process of strings.¹⁷⁾ At closer approach, strong deformation of the Y-shaped strings necessarily occurs so as to keep the Y-structure, as illustrated in Fig. 2(b). Resulting excitation energy is supposed to be considerably large.

§3. Origin of repulsive core of baryon-baryon interaction described in the String-Junction model

At present it is an open problem whether the repulsive core^{*)} exists commonly in all the BB states or changes its aspects depending on flavor and spin. At least it certainly exists in the 1S_0 and 3S_1 states in the nucleon-nucleon (NN) system. Although there is no sound evidence for its existence in the NN P -waves and higher partial waves, the assumption of its existence in all the NN states does not contradict with the experimental data. As for the systems involving hyperons (Y), there is no experimental information on specific properties of short-range NY and YY interactions. Most of theoretical studies are usually done following the NN case under the assumption of approximate flavor symmetry. At present the assumption that the repulsive core exists in all the states of the BB system does not contradict with the observations. ^{**)}

From the viewpoint of the SJM, because we consider the repulsive core as an effect originating from the confinement mechanism, the universal repulsive core (acting commonly for all the baryon pairs) is a natural consequence.

3.1. Exotic dibaryon states and junction-pair states

When two B approach very closely, the two Y-shaped strings inevitably overlap. At full overlap two B lose their identity because they cannot keep the Y-shaped structure and finally fuse. How does the confinement mechanism work at such situation? To this problem, the SJM provides us with the viewpoint that the strings and junctions make the closed net of the color flux-tubes with six quarks at the ends by producing the exotic dibaryon state, which is illustrated in the (left) of Fig. 3. As it has $N_q = 6$ and $N_J = 4$, we denote it by D_6^4 . ^{***)}

In this exotic dibaryon state D_6^4 , there appear three J and one \bar{J} , and the three interjunctions connecting them, which are absent in the ordinary hadrons, manifest the exotic character. Because the reality of J and consequently of \bar{J} becomes firm, the appearance of the oriented interjunction connecting them (indicated by the arrow emerging from J and ending \bar{J}) is naturally expected. Thus we consider the existence of D_6^4 quite reasonable. Although the arguments in the following done on the assumption of the existence of D_6^4 predicted by the SJM are speculative, we can obtain meaningful consequence. In the last two sections, after presentation of the results, we make comments on such problematic points that positive observational information on the existence of D_6^4 has not been reported yet and no lattice QCD simulation about it has been performed yet.

^{*)} There are several ways to describe the short-range repulsive effect in the BB interaction. We simply call this effect by “repulsive core” in this paper.

^{**)} If the so-called H-particle, a dibaryon state of the flavor singlet, would be observed as a spatially compact bound state, this assumption does not hold. But at present the H-particle has not been observed.

^{***)} We denote a hadron H with N_q and N_J by $H_{N_q}^{N_J}$.

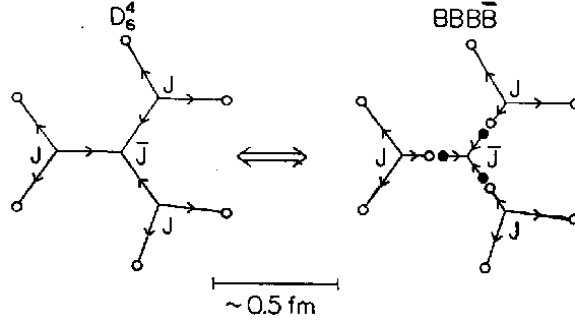


Fig. 3. (Left) String-junction structure of exotic dibaryon D_6^4 . (Right) Formation of $BBB\bar{B}$ through fission of three interjunctions.

The mass of D_6^4 is expressed with use of Eqs. (2.4) and (2.5):

$$\begin{aligned} m(D_6^4) &= 6m_q + 3m_J + m_{\bar{J}} + 3m_{IJ} = 2(3m_q + m_J) + m_J + m_{\bar{J}} + 3m_{IJ} \\ &= 2m_B + \mathcal{E}, \end{aligned} \quad (3.1)$$

$$\mathcal{E} \equiv 2m_J + 3m_{IJ}. \quad (3.2)$$

The energy of D_6^4 is higher than that of the ordinary state of $2B$ ($2m_B$) by \mathcal{E} . D_6^4 can not turn back directly to BB . In order to turn into a system of the ordinary hadrons, the cut of three interjunctions is required. In doing so, as shown in Fig. 3 (right), at the cut places the quark pairs $q\bar{q}$ are created and then the baryon-pair $B\bar{B}$ is created. As a result the system turns to $BBB\bar{B}$. Its energy is higher than $2m_B$ by about $2m_B$, and therefore we can take $\mathcal{E} \sim 2 \text{ GeV}$. *) This is simply understood from the fact that N_J increases by 2, compared with the ordinary $2B$, according to the mass formula.

In the description mentioned above, the exotic dibaryon D_6^4 is considered as to be formed through the virtual creation of the $B\bar{B}$ pair. The exotic dibaryon D_6^4 is also formed through another process. Such an example is shown in Fig. 4. In the (left) of this figure, situation prior formation is illustrated: In order for two parallel strings with the same orientation belonging to different baryons to overlap, one of them is excited by formation of the string loop accompanying the junction pair. In the (right) of the figure, situation just after formation of D_6^4 is illustrated: Two strings with opposite orientations transform into two hair-pin like strings.¹⁸⁾ The D_6^4 is surely formed, although some of the strings are strongly deformed. In this case too, since the junction number N_J increases by 2 for the formation of one junction pair, the excitation energy is about 2 GeV. Such a loop-type excitation of one string has been studied in a recent work on the pentaquark problem by Suganuma and others.^{19),20)} In lattice QCD calculations in a study of the single baryon, gluonic excitation has been shown to appear at about 1 GeV and to be regarded as a global

*) In the isolated $B\bar{B}$, meson-exchange effect works attractively mainly due to the ω and σ , but here such effect is not considered because it is uncertain in many-baryon configuration under consideration.

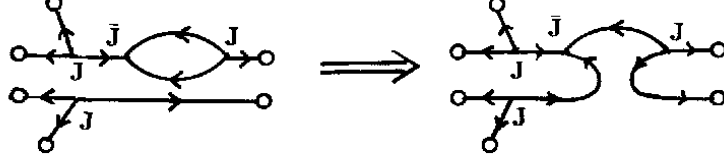


Fig. 4. Illustration of an example for another process to create D_6^4 ; (left) an excited string appears to create a string loop and a $J\bar{J}$ pair and another adjacent string is in parallel with the same orientation which is opposite to the orientation of the strings in the loop and (right) rearrangement takes place between two strings with opposite directions, leading to the formation of D_6^4 .

excitation of the whole Y-type flux-tube system.²¹⁾ On the contrary, the excitation shown in Fig. 4 is that of a single flux-tube and appears in the energy near $\mathcal{E} \sim 2$ GeV for the $B\bar{B}$ -type excitation, apart from the deformation energy of the string.

The $B\bar{B}$ pair turns into a series of states commonly involving the $J\bar{J}$ pair, as shown in Fig. 5; M_4^2 and M_2^2 (exotic mesons), and states involving no quark S_0^2 (a kind of glue balls). We note that, in later descriptions, $\mathcal{E} \sim 2$ GeV plays a role of the key number. The loop in M_2^2 created by fusion of the q and \bar{q} in M_4^2 is the same with the one appearing in Fig. 4. So we may call the excitations mentioned above simply the $B\bar{B}$ excitation or the $J\bar{J}$ excitation.

This series of states with the baryon number=0 and $N_J = 2$ contain commonly the J and \bar{J} which we call the junction-pair. Here they are denoted by the symbol \mathcal{M} . Because of irrelevance to the number of quarks, they are the states inherent in the gluon degrees of freedom. The energies of \mathcal{M} are distributed about the central value of about 2 GeV with a spread of several hundreds MeV, as far as we are concerned with u , d and s . (For details, §6.1 should be referred to.) This means that there exist a group of states with the baryon number=2, consisting of D_6^4 and $[B\bar{B} + \mathcal{M}]$, in the energy region higher by about 2 GeV above the ordinary $B\bar{B}$.

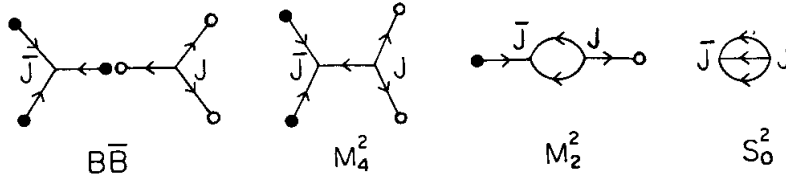


Fig. 5. A series of states with the baryon number =0 formed by fusion of q and \bar{q} from $B\bar{B}$, which we call junction-pair states, denoted as \mathcal{M} .

3.2. Repulsive core of the BB interaction

We can paraphrase the statement mentioned above as follows. In order for the confinement mechanism to persist when two baryons overlap fully, it is necessary to form the string-junction net where the arrows attached to the oriented strings are closed. It requests the excitation energy of $\mathcal{E} \sim 2$ GeV, which makes the creation of $B\bar{B}$ pair (more generally the junction pair) possible. If the kinetic energy of BB in the center of mass system is much smaller than m_B (e.g., in the elastic scattering region), the system is apt to avoid such a large excitation. The potentiality that two baryons reside apart to get rid of such expense is equivalent to the existence of a high potential barrier between two baryons, namely, the repulsive core. Its height is of the order of $\mathcal{E} \sim 2$ GeV, and its range is the relative distance where two baryons begin to overlap largely.

The potential mentioned above may be defined in a sense of the adiabatic approximation usually adopted to get a static potential, where the relative kinetic energy of two baryons is taken into account in solving the Schrödinger equation. In principle, we should treat the energy described in the quark-gluon level of QCD, but here we use the approximate value obtained in the SJM, abbreviated as $E^{(SJM)}$. We define an interaction potential dominating in the core region to be used in the baryon level, by the difference between the energies $E^{(SJM)}$ at two relative distances of BB , $r_{12} = |\mathbf{r}_1 - \mathbf{r}_2|$ and $r_{12} \rightarrow \infty$, where \mathbf{r}_1 and \mathbf{r}_2 are the positions of two baryons:

$$V^{(\text{core})}(r_{12}) \equiv E^{(SJM)}(r_{12}) - E^{(SJM)}(r_{12} \rightarrow \infty). \quad (3.3a)$$

For the SJM, since we can give the values of $E^{(SJM)}$ only at the two limiting cases, at $r_{12} \ll \ell_s$ and $r_{12} \gg \ell_s$, compared with the string length of the ordinary baryon $\ell_s \simeq 0.5$ fm, the magnitude of $V^{(\text{core})}$ at closest approach is obtained as

$$V^{(\text{core})}(r_{12} \ll \ell_s) \simeq E^{(SJM)}(r_{12} \ll \ell_s) - E^{(SJM)}(r_{12} \gg \ell_s) \quad (3.3b)$$

$$\simeq E(D_6^4) - 2m_B \simeq 2m_B + \mathcal{E} - 2m_B \simeq \mathcal{E}. \quad (3.3c)$$

This $V^{(\text{core})}$ is regarded as the interaction of short range in the hadron level, and is incorporated into the two-baryon potential, together with the meson-exchange potentials of the long and intermediate range.

The view that the core-like short-range repulsion is a latent effect reflecting the energy needed for the overlap is quite natural because it is familiar in the atomic and nuclear systems. The repulsive core between two atoms (typically between two closed-shell atoms), which is often approximated by a hard-sphere potential, is understood as originating from the large excitation energy of electron states which is needed for the overlap of the electron clouds. This repulsive core is “strong”; it is so steep that its core height becomes by several order of magnitude larger than the strength of attractive pocket just outside the core, if a soft core potential is used. The repulsive core used in the effective potential between two nuclei (typically between two light closed-shell nuclei, e.g., between two α -particles) is understood as originating from the excitation energy of nucleon states required for overlap of two nuclei. This repulsive core is “moderately strong”, as its height of ~ 150 MeV is larger by one order of magnitude than the depth of attractive pocket (~ 10 MeV).

for the $\alpha - \alpha$ case, although a hard core potential may be used phenomenologically for low energy scattering. ^{*)} The BB repulsive core of the ~ 2 GeV height with the ~ 0.1 GeV attractive pocket (in the S -wave) belongs to the “moderately strong” class.

These features of the repulsive core in the atomic and nuclear systems are attributed to the effects of the Pauli-exclusion principle. The related internal degrees of freedom is the spin of the electron in the atomic system and the spin-isospin of the nucleon in the nuclear system. As for the repulsive core of the BB system, the related internal degrees of freedom is the color. Although we see similarity among three cases, it is to be noted that the confinement mechanism is essential in the BB case, and the exclusion principle relevant to the color degrees of freedom alone is not essential because the quarks have many internal degrees of freedom.

The range of the repulsive core originating from the confinement mentioned above is smaller than the relative distance where two Y-shaped structures begin to touch. If the string length of the Y-shape in the ground state of B is taken as $\ell_s \simeq 0.5$ fm corresponding to $L_{\min} \simeq 1.5$ fm in Eq. (2.2), the half overlap of two B corresponds to the relative distance $\simeq 0.5$ fm. Thus the range of about 0.5 fm is a reasonable choice for the repulsive core derived in the SJM. In a previous work in 1964,²²⁾ where we considered “structural core” of the NN interaction from the viewpoint of the composite model of hadrons, we have chosen the shape of the NN repulsive core as the gaussian form, implying that the repulsion grows as the overlap of two nucleons becomes large. On the same standpoint, the present author constructed a realistic NN potential with the gaussian repulsive core, called OPEG.²³⁾ In a representative version of OPEG potential, OPEG-A, the strength and the range of the repulsive core are taken as

$$\mathcal{V}_C(r) = \mathcal{V}_C^0 \exp [-(r/\eta_C)^2], \quad \text{with } \mathcal{V}_C^0 = 2 \text{ GeV}, \quad \eta_C = (0.45 - 0.5) \text{ fm}. \quad (3.4)$$

The repulsive core $V^{(\text{core})}$ obtained in the SJM has the feature well corresponding to \mathcal{V}_C of OPEG-A. Thus we can understand the origin of the BB repulsive core in the SJM, as the manifestation of the confinement mechanism in the BB system.

In the notion of the state vector, the present context is interpreted as follows. The state vector of the system $|\Psi\rangle$ is a superposition of the part spanned by the states of two ordinary baryons, denoted as $P|\Psi\rangle$, and the part spanned by the D_6^4 plus $[BB + \mathcal{M}]$ states, denoted as $Q|\Psi\rangle$, where P and Q are the projection operators ($P+Q=1$). The latter states $Q|\Psi\rangle$, whose existence is limited to the core region and at high excitation, has the latent influence on the former state $P|\Psi\rangle$. Such influence appears as suppression of the BB relative wavefunction at very small distances, corresponding to the effects by the repulsive core.

It is not simple to show logical connection between the core-like repulsion shown in the adiabatic approximation and the effective BB interaction described in the $P|\Psi\rangle$

^{*)} This repulsive soft core in the potential form is understood as a reflection of the forbidden relative states which should vanish under the Pauli-exclusion principle. Such a feature is described by using the orthogonality condition to the forbidden relative states or by using a strong nonlocal potential.

influenced by the existence of $Q|\Psi\rangle$. First of all we notice that substantial effects appear in two ways. One is the effect of the orthogonality between $P|\Psi\rangle$ being in the scattering state and $Q|\Psi\rangle$ with spatially compact structure and high excitation energy. The other is the channel coupling effect. The resulting core-like repulsion in the adiabatic approximation means that the former overwhelms the latter. The reasoning is as follows.

1. *Effect of the orthogonality*

In the scattering problem, as the energy is fixed to the incident energy, the observable effect appears in scattering phase shifts in the asymptotic region. In the problem under consideration, the influence from $Q|\Psi\rangle$ is rather special, because its existence is limited in the inner-most region of the BB relative distance (core region), $r \lesssim b \simeq 0.5$ fm. We notice that the primary influence from the orthogonality to $Q|\Psi\rangle$ brings about the nodal behavior of the BB relative radial wavefunction of $P|\Psi\rangle$ at the core region, which is to be described by a strong nonlocal interaction. The reason is that such radial nodes do not mean attraction to produce bound states but the almost energy-independent nodal behavior gives repulsive effects, because the amplitude of the oscillatory wavefunction is suppressed and the outermost node plays a role equivalent to the boundary condition given by a core-like repulsion with the radius at $r \simeq b$, in off-resonance situation.^{*)} If there exist many such states of $Q|\Psi\rangle$, the wavefunctions are strongly suppressed in the whole core region and the situation becomes as if the hard core exists. If some attractive potential works outside the core region, even though the scattering phase shifts show attractive feature at low energies, repulsive feature appears at high energies because repulsive-core effects are strong enough to overwhelm the attractive effects. The repulsive core is to be considered as a simple local potential form in such a limited sense that it represents only the suppressed feature of the wavefunction given by the nonlocal interaction in the core region mentioned above. ^{**)}

2. *Channel coupling effect*

In addition there appears a dynamical effect due to the channel coupling through the interaction matrix elements (i.m.e.) between $P|\Psi\rangle$ and $Q|\Psi\rangle$. Generally the channel coupling effect is considered to give attractive effect on the channel with a lower energy, in ordinary situation of bound states. Since the states of $P|\Psi\rangle$ and $Q|\Psi\rangle$ under consideration have substantially different structures with the large energy difference, the i.m.e. are not large, and the channel coupling effect seems moderate. We cannot enter further into this problem, because of lack of concrete knowledge about the wavefunctions of $Q|\Psi\rangle$ in the SJM.

We can summarize the results in the SJM approach to the BB short-range interaction, by saying that the repulsive core preventing two baryons from close approach

^{*)} Previously we showed such an example in the $NN\ ^1S_0$ scattering.^{24), 25)}

^{**)} We have encountered such problem concerning interaction between nuclei: The strong non-local interaction giving rise to the inside oscillatory wavefunction appears when we treat the inter-nucleus coordinate as a dynamical variable with use of the resonating group method, while the repulsive core with literal meaning appears when we treat the inter-nucleus distance as a variational parameter with use of the generator coordinate method.^{26), 27)}

appears as the latent effect on the two-baryon states, corresponding to the high excitation energy needed for the system to fully overlap.

§4. Short-range three-body repulsion in the *BBB* system in the string-junction model

In this section, with use of the SJM, at first we seek main aspects of three-baryon interaction (3BI) originating from the confinement, next give its potential form and strength and then derive a density-dependent effective two-body potential which is used to obtain its contributions to the energy of baryonic matter. Before entering individual steps, we briefly describe the treatment of 3BI adopted here.

4.1. Outline of treatment of three-baryon interaction in baryonic matter

The Hamiltonian, which enables us to understand properties of nuclei and baryonic matter, is expressed in the nonrelativistic treatment as follows:

$$H = \sum_i K_i + \sum_{i < j} V(i, j) + \sum_{i < j < k} V^{(3B)}(i, j, k) + \cdots, \quad (4.1)$$

where K_i is the kinetic energy of the i -th baryon B_i (including the rest mass difference from the nucleon mass), V two-baryon interaction and $V^{(3B)}$ three-baryon interaction. The ellipsis means four-body (and higher) interactions, which are neglected. We take the following form for $V^{(3B)}(i, j, k)$ consisting of the cyclic sum:

$$V^{(3B)}(i, j, k) = \sum_{\text{cyclic}} W(i, j; k) = W(i, j; k) + W(j, k; i) + W(k, i; j), \quad (4.2)$$

where $W(i, j; k)$ represents the 3BI which arises additionally from the approach of the k -th baryon to the pair of B_i and B_j . Such a form is familiar, for example, in the study of the three-nucleon interaction from the two-pion exchange process via Δ -excitation of the 3rd nucleon N_3 intervening between two nucleons (N_1 and N_2), initiated by Fujita and Miyazawa.²⁸⁾ (We take $i = 1$, $j = 2$, $k = 3$ as the representative one without loss of generality.) Also the repulsive three-nucleon interaction introduced phenomenologically by Phandharipande and others has such a form.^{29),30)} As shown below, this form is suitable for the 3BI derived in the SJM.^{*)} A primary problem is how to describe a representative one of the SJM type, $W(1, 2; 3)$.

Once $W(1, 2; 3)$ is obtained in the SJM, next we derive the effective two-body potential by taking the sum of the diagonal matrix elements over the occupied states of the 3rd baryon B_3 in the baryonic matter. This effective two-body potential importantly depends on the baryon density ρ , and we denote it by $U(1, 2; \rho)$. At this

^{*)} The cyclic sum is naturally understood by viewing the situation shown in Fig. 6 of §4.2. The 3BI under consideration in the SJM is regarded as arising through the virtual creation of one $B\bar{B}$ pair in B_3 approaching to the two baryons $B_1 B_2$ near by. The formal structure in this case is analogous to the case of Fujita-Miyazawa type three-nucleon interaction, if the Δ -excitation in the latter is supposed to be replaced by the one $B\bar{B}$ pair excitation in the former. The expression of Eq. (4.2) implies a wider situation and is applicable to other cases not restricted to the cases mentioned above.

step we take into account the effects of the short-range correlation (abbreviated as SRC) coming from the interactions between $B_1 - B_3$ and between $B_2 - B_3$, while the SRC between $B_1 - B_2$ is included in the final step of the energy calculation. Then we can calculate the energy of the baryonic matter E in the framework of the reaction matrix theory, by adopting $V(i, j) + U(i, j; \rho)$ as the two-body potential. In this way the symmetric sum of $V^{(3B)}(i, j, k)$ in Eq. (4.2) is automatically included in the sum over all the two-baryon pairs. The interaction energy E_{int} is given by

$$E_{\text{int}} = \frac{1}{2} \sum_{\alpha, \beta}^{\text{occupied}} \langle \phi_{\alpha}(1) \phi_{\beta}(2) | V(1, 2) + U(1, 2; \rho) | \psi_{\alpha, \beta}(1, 2) - \psi_{\alpha, \beta}(2, 1) \rangle, \quad (4.3)$$

where $\phi_{\alpha}(1) \phi_{\beta}(2)$ is the unperturbed (plain-wave) function of the baryon pair and $\psi_{\alpha, \beta}(1, 2)$ is the pair wave function in the matter which includes two-body correlations. In addition to the contribution from the usual two-body potential V , the effects of the 3BI in the SJM is given as the contribution from the effective two-body potential U .

4.2. Exotic tribaryon states in the BBB system and strength of three-baryon force

Here in the SJM, we study the representative part of the 3BI, $W(1, 2; 3)$, which arises additionally when the 3rd baryon B_3 approaches to the adjacent two baryons (B_1 and B_2). We show that a short-range three-body repulsion originates from the confinement mechanism, by extending the reasoning in deriving the universal repulsive core of the BB interaction in the previous section to the three-baryon system.

In order for baryons to overlap fully, from the viewpoint of the SJM, the strings and junctions representing color flux-tubes should be rearranged to hold the confinement in such situation. In the BB system, it becomes possible by creating one $B\bar{B}$ pair. Here we consider the extension of this idea to the BBB system by taking two steps. (1) We imagine the situation that B_1 and B_2 are close enough to form the exotic dibaryon state $D_6^4(1, 2)$ and the third baryon B_3 approaches to $D_6^4(1, 2)$, as illustrated in Fig. 6(a). (2) One $B\bar{B}$ pair is virtually created between them as

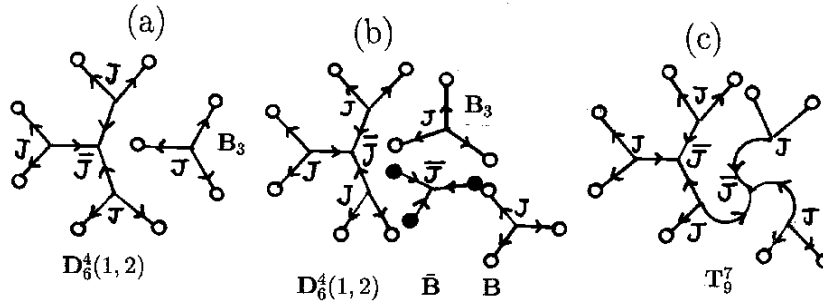


Fig. 6. Illustration of formation of the exotic tribaryon in two steps; (a) approach of B_3 to the exotic dibaryon $D_6^4(1, 2)$ formed by adjacent B_1 and B_2 , (b) virtual creation of one $B\bar{B}$ pair between them, and (c) formation of the exotic tribaryon T_9^7 after the fusion of the strings of q and \bar{q} .

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in Fig. 6(b), and after the fusion of the strings of q and \bar{q} the system turns to the connected string-junction net with $N_q = 9$ and $N_J = 7$ with increase of three interjunctions instead of three $q\bar{q}$ -pair strings, as illustrated in Fig. 6(c). This state is the exotic tribaryon state denoted by T_9^7 .

The energy of T_9^7 is given using the mass formula as follows:

$$\begin{aligned} m(T_9^7) &= 9m_q + 5m_J + 2m_{\bar{J}} + 6m_{IJ} = 3(3m_q + m_J) + 2m_J + 2m_{\bar{J}} + 6m_{IJ} \\ &= 3m_B + 4m_J + 6m_{IJ} = 3m_B + 2\mathcal{E}. \end{aligned} \quad (4.4)$$

The mass $m(T_9^7)$ is higher by $2\mathcal{E} \sim 4$ GeV than the energy of the ordinary BBB system, where \mathcal{E} is defined in Eq. (3.2). In this energy, its half ($\mathcal{E} \sim 2$ GeV) is attributed to the formation of $D_6^4(1, 2)$. Therefore the energy arising additionally from the approach of B_3 is $\mathcal{E} \sim 2$ GeV. This is clear, because the $B\bar{B}$ is created and the junction number increases by 2. Next we consider the 3BI based on the SJM, whose strength is this additional increase of energy ($\mathcal{E} \sim 2$ GeV) at the closest approach of B_3 to $D_6^4(1, 2)$.

We write the 3BI in the SJM (abbreviated as SJM-3BI), following the reasoning adopted in the two-baryon case. A representative part of the SJM-3BI denoted by $W(1, 2; 3)$ is given as the difference between the energies at the two situations taken along the “path” which leads to the formation of T_9^7 at the close approach of B_3 to the exotic dibaryon $D_6^4(1, 2)$. These energies are to be described in the quark-gluon level of QCD, in principle, but here we use the approximate values obtained in the SJM, denoted by $E^{(SJM)}(\xi_3; D_6^4(1, 2))$, where ξ_3 is the relative distance between the position of B_3 and that of the center of mass of D_6^4 . We define the magnitude of

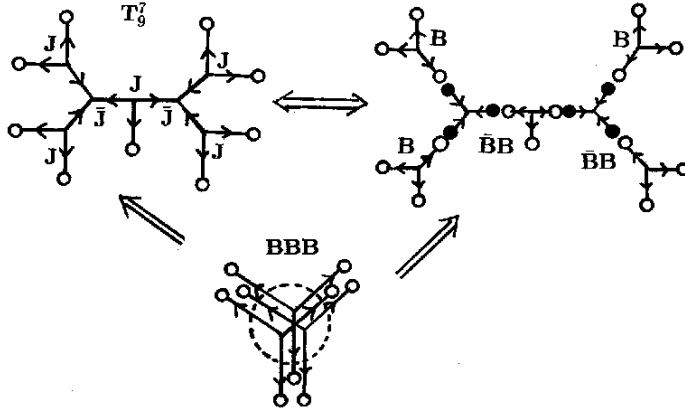


Fig. 7. Pictorial view of the exotic tribaryon and its preformation stage; (left) string-junction net of the tribaryon T_9^7 , (right) $BBB(\bar{B}\bar{B})(B\bar{B})$ states arising from the fission of the interjunction strings, and (bottom) an example illustrating one of many possible configurations for full overlap of BBB , where the dotted area indicates the formation region of a string-junction net. Such view is the unfolded-sheet drawing of the tribaryon having three-dimensional spread.

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$W(1, 2; 3)$ as

$$W(1, 2; 3) \equiv E^{(SJM)}(\xi_3; D_6^4(1, 2)) - E^{(SJM)}(\xi_3 \rightarrow \infty; D_6^4(1, 2)), \quad (4.5a)$$

For the SJM, since we can give the values of $E^{(SJM)}$ only at the two limiting situations, namely, those at $\xi_3 \ll \ell_s$ and $\xi_3 \gg \ell_s$ (equivalent to $\xi_3 \rightarrow \infty$), compared with the string length of the ordinary baryon $\ell_s \simeq 0.5$ fm, the magnitude of $W(1, 2; 3)$ at the closest approach of B_3 is obtained as

$$\begin{aligned} W(1, 2; 3) &\simeq E^{(SJM)}(\xi_3 \ll \ell_s; D_6^4(1, 2)) - E^{(SJM)}(\xi_3 \gg \ell_s; D_6^4(1, 2)) \quad (4.5b) \\ &\simeq E(T_9^7) - (m_B + E(D_6^4)) \simeq 3m_B + 2\mathcal{E} - (m_B + 2m_B + \mathcal{E}) \simeq \mathcal{E}, \quad (4.5c) \end{aligned}$$

which is considered as the strength of the SJM-3BI.

$W(1, 2; 3)$ regarded as the interaction in the hadron level is a genuine 3BI in the following sense. In $D_6^4(1, 2)$, the original baryons (B_1 and B_2) have lost their identity of the baryon as shown in Fig. 6(a), although $D_6^4(1, 2)$ is formed from B_1 and B_2 through the virtual creation of the $B\bar{B}$ pair. Therefore, the interaction between $D_6^4(1, 2)$ and B_3 in the configuration leading to the formation of T_9^7 is an intrinsic three-body effect which is not reduced to any effect due to the two-body interaction.

The state vector of the system $|\Psi\rangle$ can be written, with use of two projection operators P and Q satisfying $P + Q = 1$, as the sum of the state vector $P|\Psi\rangle$ which describes the states of three ordinary baryons and the state vector $Q|\Psi\rangle$, which describes the states arising from $D_6^4(i, j)$ plus B_k and leading to T_9^7 (denoted by $Q_T|\Psi\rangle$) and the higher states with larger junction numbers. Restricting $Q|\Psi\rangle$ to $Q_T|\Psi\rangle$, we can write Q_T as $Q_T = Q_3 + Q_1 + Q_2$, where Q_3 projects out the states brought about along the “path” leading to T_9^7 formed from $D_6^4(1, 2)$ and the intervener B_3 . Q_1 and Q_2 are given by its cyclic permutations. This means that three $Q_k|\Psi\rangle$ spans different Fock spaces. The reason is that, for $k = 3$, once $D_6^4(1, 2)$ is formed from B_1 and B_2 , the simultaneous formation of $D_6^4(2, 3)$ and/or $D_6^4(3, 1)$ does not take place, because such a double (also triple) D_6^4 configuration cannot form a closed net of the oriented strings and junctions. The latent effect from the existence of $Q_3|\Psi\rangle$ on the three-baryon state $P|\Psi\rangle$ appears as the repulsive 3BI, $W(1, 2; 3)$. In the same way, combination of $D_6^4(2, 3)$ and B_1 ($D_6^4(3, 1)$ and B_2) leads to $Q_1|\Psi\rangle$ ($Q_2|\Psi\rangle$), which brings about $W(2, 3; 1)$ ($W(3, 1; 2)$) on $P|\Psi\rangle$. Thus the sum, $Q_T|\Psi\rangle = \sum_1^3 Q_k|\Psi\rangle$, provides the SJM-3BI as the latent effect on $P|\Psi\rangle$, which is given by the cyclic sum of $W(i, j; k)$ as in Eq. (4.2).

The exotic tribaryon states can be attained also in one step from the ordinary BBB system by creating two $B\bar{B}$ pairs as illustrated in the (right) of Fig. 7. Fusion of the strings of q and \bar{q} leads to the interjunctions, and the $BBB(B\bar{B})(B\bar{B})$ system turns into the exotic tribaryon T_9^7 , shown in the (left) of Fig. 7. This state is the same with the one denoted by A_9 in ref. 11b).

4.3. Effective two-body potential from three-baryon interaction in the baryonic matter

In order to evaluate the effect of the SJM-3BI in the baryonic matter, we employ a practical way, called the effective two-body potential (2BP) method, which enables

us to calculate the effects from 3BI in a form of the additional contribution to the two-baryon potential. This method was developed by Hokkaido group in the study on the role of the 3BI due to the two-pion exchange via Δ -excitation in nuclear matter,³¹⁾ and used in recent works on 3BI.^{32),33)} This is a way to obtain the effects of 3BI approximately, in place of full scale Bethe-Faddeev calculation with 3BI. The procedure in calculating the effective 2BP from the SJM-3BI are as follows.

1. First of all, effects of the 3BI appear under the influence of the primary interaction, the two-baryon interaction V , that is, its main effects are obtained with use of the wavefunction distorted by V . In baryonic matter, the distortion from the plane wave is remarkable at small inter-baryon distances, and is called the short-range correlation (abbreviated as SRC).
2. Applying the method, we derive an effective 2BP, $U(1, 2; \rho)$, from $W(1, 2; 3)$ acting in the baryonic matter, by taking the sum of the diagonal matrix elements of (i.e., by taking the average) over the occupied states of the 3rd baryon B_3 .
3. By the procedure of taking this average, because the states of B_3 do not change, the effect from $W(1, 2; 3)$ results in the additional effect on the interaction between B_1 and B_2 .
4. A remaining problem is to choose a functional form of $W(1, 2; 3)$ with use of three spatial variables (\mathbf{r}_1 , \mathbf{r}_2 and \mathbf{r}_3), which are available here because of the universality (independence on flavor and spin) of the SJM-3BI.

$$\begin{aligned}
 U(1, 2; \rho) &\equiv \int d\mathbf{r}_3 \sum_{\gamma}^{(\text{occ})} \langle \phi_{\gamma}(3) | W(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_3) | \phi_{\gamma}(3) \rangle f^2(\mathbf{r}_1 - \mathbf{r}_3) f^2(\mathbf{r}_3 - \mathbf{r}_2), \\
 &= \rho \int d\mathbf{r}_3 W(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_3) f^2(\mathbf{r}_1 - \mathbf{r}_3) f^2(\mathbf{r}_3 - \mathbf{r}_2),
 \end{aligned} \tag{4.6}$$

where $f(\mathbf{r}_i - \mathbf{r}_j)$ is a SRC function which includes the suppression of the relative wavefunction by the repulsive core for $B_i - B_j$ pair. Here we assume W being local. $\phi_{\gamma}(3)$ is the plane wave state, whose spatial part is $\phi_{\gamma}(3) = \exp(i\mathbf{q}_{\gamma} \cdot \mathbf{r}_3)/\sqrt{\Omega}$, Ω being the normalization volume. The sum over the states ϕ_{γ} simply gives the baryon density ρ , because of the universality of the SJM-3BI.

The same procedure is applied to $W(2, 3; 1)$ and $W(3, 1; 2)$ leading to the effective 2BPs in the baryonic matter, $U(2, 3; \rho)$ and $U(3, 1; \rho)$, respectively.

For choice of the functional form of $W(1, 2; 3)$, at best, we can take into account its property that $W(1, 2; 3)$ is the interaction mediated by B_3 between B_1 and B_2 and is of short-range nature. In this context we take the simplest form, the product of a function of $B_1 - B_3$ distance and a function of $B_2 - B_3$ distance, as follows.

$$W(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_3) \equiv W_0 g(\mathbf{r}_1 - \mathbf{r}_3) g(\mathbf{r}_2 - \mathbf{r}_3), \tag{4.7}$$

where g is a function such that $g(r_{ij} \rightarrow 0) \rightarrow 1$ and vanishes at $r_{ij} \gg \ell_s$ and $W_0 \sim \mathcal{E}$. Here, for convenience of calculations, we choose a gaussian form for $g(r_{ij})$ with the range of $\eta_C = 0.45 - 0.50$ fm in Eq. (3.4).

$$g(\mathbf{r}_i - \mathbf{r}_j) = \exp(-\lambda r_{ij}^2), \quad \lambda \simeq \frac{1}{\eta_C^2} \simeq (4.9 - 4.0) \text{ fm}^{-2}. \tag{4.8}$$

Denoting the Fourier transform of $g(\mathbf{r})$ as $G(\mathbf{q})$,

$$g(\mathbf{r}) = \frac{1}{(2\pi)^3} \int d\mathbf{q} G(\mathbf{q}) e^{i\mathbf{q}\cdot\mathbf{r}}, \quad (4.9)$$

$$G(\mathbf{q}) = 4\pi \int dr r^2 e^{-\lambda r^2} j_0(qr) = (\pi/\lambda)^{3/2} e^{-q^2/4\lambda}. \quad (4.10)$$

After the transform of momentum variables, $U(1, 2; \rho) = U(\mathbf{r}_{12}; \rho)$ is expressed, as follows:

$$U(\mathbf{r}_{12}; \rho) = \frac{\rho W_0}{(2\pi)^3} \int d\mathbf{q} e^{-i\mathbf{q}\cdot(\mathbf{r}_1 - \mathbf{r}_2)} \int d\mathbf{q}_1 h(\mathbf{q} - \mathbf{q}_1) G(\mathbf{q}_1) \int d\mathbf{q}_2 h(\mathbf{q} - \mathbf{q}_2) G(\mathbf{q}_2). \quad (4.11)$$

Here $h(\mathbf{q})$ is the Fourier transform of the SRC function squared f^2 :

$$f^2(\mathbf{r}) \equiv \int d\mathbf{q} h(\mathbf{q}) e^{-i\mathbf{q}\cdot\mathbf{r}}. \quad (4.12)$$

It is to be noted that $U(\mathbf{r}_{12}; \rho)$ is strongly density-dependent.

Without the SRC, we have $h(\mathbf{q}) = \delta(\mathbf{q})$, and then the integral becomes

$$U_0(\mathbf{r}_{12}; \rho) = \frac{\rho W_0}{(2\pi)^3} \int d\mathbf{q} e^{-i\mathbf{q}\cdot\mathbf{r}_{12}} (G(\mathbf{q}))^2 \quad (4.13)$$

$$= \rho W_0 (\pi/2\lambda)^{3/2} \exp(-\lambda r^2/2). \quad (4.14)$$

Hereafter we write \mathbf{r}_{12} simply as \mathbf{r} .^{*)}

Now we calculate the effective 2BP (U). To take into account the SRC for B_1 - B_3 and for B_2 - B_3 , we calculate the two integrals with the same form in Eq. (4.11). In doing so we need the SRC function f extracted from the solution of the reaction matrix equation in nuclear matter.

The SRC functions change state by state. At low density of neutron matter, as the 1S_0 wave contributes mainly, we can use the SRC function of this wave denoted as $f_S(r)$. As the density goes higher, this approximation is not allowed, and we should add both the contributions from 1S_0 wave and $^3P_{J=0,1,2}$ wave. The latter depends on J due to the effects from the noncentral potentials. For energy quantities, the contributions appear in the weighted average over $^3P_{J=0,1,2}$, which is proportional to $(2J+1)$. We define the 3P -wave SRC function:

$$f_P(r) \equiv \{f(r; ^3P_{J=0}) + 3f(r; ^3P_{J=1}) + 5f(r; ^3P_{J=2})\}/9. \quad (4.15)$$

As the 1S_0 -wave and the average 3P wave contribute with the statistical weight relevant to the spin, we define the weighted average of the SRC function as

$$f_{\text{wa}}(r) \equiv \{f_S(r) + 3f_P(r)\}/4. \quad (4.16)$$

^{*)} Making estimate of U_0 , we have $U_0(r; \rho) = 84(\rho/\rho_0) e^{-2(r/\text{fm})^2}$ MeV for $\lambda = 4\text{fm}^{-2}$ and $W_0 = 2$ GeV. This value is (5-3) times larger than that with SRC at $r \simeq (0-1)$ fm. So the SRC is important for calculating effects of three-body force.

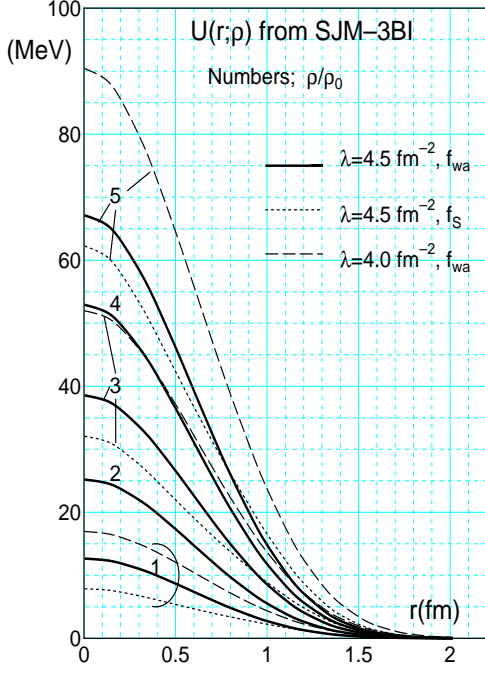


Fig. 8. Effective two-body potential $U(r; \rho)$ from universal repulsion of SJM-3BI. Numbers attached to the curves show ρ/ρ_0 . Parameters of the force range λ and the SRC function f are indicated in the figure.

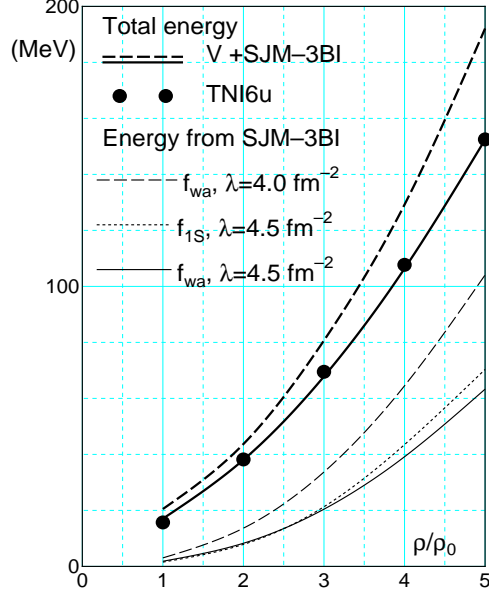


Fig. 9. Density dependence of effects from SJM-3BI on the energy/baryon in neutron matter. The lower three curves show contributions from SJM-3BI. The upper two curves show the total energy/baryon in neutron matter including SJM-3BI, where two-body (V) contribution and EOS of TNI6u (filled circles) are taken from ref. 9).

These SRC functions are constructed using the wave functions obtained for OPEG-A potential²³⁾ in the reaction matrix equation in the neutron matter.

In numerical calculations, we obtain the effective two-body potential from the SJM-3BI, using $f_S(r)$ or $f_{wa}(r)$. The damping of $f_S(r)$ at small distance is strong, but that of $f_P(r)$ is moderate. Also the damping of $f_{wa}(r)$ is not so strong.

We denote the Fourier transform G modified by the SRC effect by $G_{\text{SRC}}(q)$:

$$G_{\text{SRC}}(q) \equiv \int d\mathbf{q}_1 h(\mathbf{q} - \mathbf{q}_1) G(\mathbf{q}_1) \quad (4.17)$$

$$= 4\pi \int_0^\infty dr r^2 f_L^2(r) e^{-\lambda r^2} j_0(qr), \quad (4.18)$$

where h is substituted by its Fourier transform $h(\mathbf{q}) = \int d\mathbf{r} f_L^2(r) \exp(i\mathbf{q} \cdot \mathbf{r}) / (2\pi)^3$. $G(q)_{\text{SRC}}$ differs from $G(q)$ by including the SRC factor $f_L^2(r)$ in the integral. Then the effective two-body potential is given as

$$U(r; \rho) = \frac{\rho W_0}{2\pi^2} \int_0^\infty dq q^2 j_0(qr) \{G_{\text{SRC}}(q)\}^2. \quad (4.19)$$

In this paper the strength of the SJM-3BI is taken as $W_0 = 2$ GeV. Results of numerical calculations for $U(r; \rho)$ are shown in Fig. 8 at the density $\rho/\rho_0 = 1 - 5$, where $\rho_0 = 0.17 \text{ fm}^{-3}$ is the nuclear density. The effective two-body potential obtained from the SJM-3BI is a universal repulsion with the range of about 1 fm. Although it is of the order of 10 MeV around the nuclear density ρ_0 , it grows almost linearly as ρ goes high. It is rather sensitive to the spread of the function $g(r)$; U becomes stronger for a wider spread. The $U(r; \rho)$ obtained from the repulsive term of phenomenological 3BI used by Illinois group²⁹⁾ has a longer range and a weaker strength than those here shown.

Adequacy of the 3BI obtained here should be examined in the energy properties of nuclei. We have studied the saturation curve of the symmetric nuclear matter, including this SJM-3BI in addition to a realistic two-nucleon potential with a repulsive core and the pionic 3BI of the Fujita-Miyazawa type, and obtained a reasonable saturation curve.³⁴⁾

§5. Effects on the equation of state of neutron matter and neutron star matter

We calculate the effects from the SJM-3BI on the EOS (equation of state), energy/baryon, in neutron matter and then discuss the effects on the EOS in neutron star matter. As the SJM-3BI is universal, its first order effect in the perturbation (without the effects of the two-body correlation in Eq. (4.3)) is simply given as

$$\frac{E_U^{(1)}}{N} = \frac{k_F^3}{6\pi} \int_0^{k_F} dk P(k) \{ 4 \int dr r^2 U(r; \rho) - 2 \int dr r^2 U(r; \rho) j_0(2kr) \}, \quad (5.1)$$

where N is the baryon number (here the neutron number), k_F the Fermi momentum and the 1st (2nd) term in the curly bracket is the direct (exchange) contribution. The function $P(k)$ is the probability function of the relative momentum k , normalized as $\int_0^{k_F} dk P(k) = 1$

$$P(k) = \frac{24k^2}{k_F^3} \{ 1 - (3/2)(k/k_F) + (1/2)(k/k_F)^3 \}. \quad (5.2)$$

The two-body correlation between B_1 and B_2 including the SRC effects are taken into account by multiplying $f_S(r)$ on the 1S_0 wave and $f_P(r)$ on the 3P wave. Then the contribution to the energy/baryon from the SJM-3BI (the part of the U -term in Eq. (4.3)) is given by

$$\begin{aligned} \frac{E_U}{N} = & \frac{k_F^3}{6\pi} \int_0^{k_F} dk P(k) \{ \int dr r^2 U(r; \rho) (f_S(r) + 3f_P(r)) \\ & + \int dr r^2 U(r; \rho) (f_S(r) - 3f_P(r)) j_0(2kr) \}. \end{aligned} \quad (5.3)$$

Since the effective two-body potential U has the density dependence being almost linear, the growth of E_U/N is stronger than the linear density-dependence. The total

energy /baryon is given by the sum of the contribution from the two-body potential V and the contribution from the repulsive SJM-3BI (E_U/N), as shown in Fig. 9. For the former we use the result by Takatsuka et al.⁹⁾ The upper two curves show the total energy/baryon (EOS at zero temperature). The bold solid curve is for the case of the following parameters; the range of the function g , $\lambda = 4.5 \text{ fm}^{-2}$ (corresponding to the spread $\simeq 0.47 \text{ fm}$), $W_0 = 2 \text{ GeV}$ and the f_{wa} for the SRC function. In this case we obtain almost the same EOS with TNI6u (shown by \bullet in the figure)⁹⁾ without adjustment of parameters. The predicted maximum mass of neutron stars is $M_{\text{max}} \simeq 1.8M_{\odot}$, which is consistent with the observations. If we use a little wider range parameter, $\lambda = 4.0 \text{ fm}^{-2}$ (corresponding to the spread=0.5 fm), we can provide the stiffer EOS shown by the upper bold-dashed curve, which leads to $M_{\text{max}} \simeq 2.0M_{\odot}$. A full account concerning the EOS including the meson exchange 3BI will be reported elsewhere.³⁵⁾

Because this SJM-3BI is independent of flavor and spin, its repulsive effect is not influenced by the hyperon-mixing in the neutron star matter. Thus we can get rid of the dramatic softening of EOS, which otherwise leads to the contradiction with the observations.

§6. Discussion

Here we discuss on several related points.

6.1. On the parameters in the SJM

In the arguments in the previous sections, we preferentially use the key number $\mathcal{E} \sim 2 \text{ GeV}$, without direct use of the parameters in the SJM. Here we try to estimate the values of the parameters (m_q , m_J and m_{IJ}) in the mass formula of the SJM, utilizing this number.

Although m_B (the mass of the single baryon B) is in considerable variation, its source is considered as mainly due to the difference in current masses of involved quarks. From the viewpoint of the SJM, since the binding mechanism is mainly determined by the strings and junctions, we have a ground for the statement that the baryon mass difference is mainly attributed to the current quark masses, if we disregard the effects of spin-dependent interactions such as the color magnetic interaction leading to the color hyperfine splitting. In the following we take the current quark mass $\simeq 0$, suitable for the u and d quarks. Thus we take $m_B \simeq 1 \text{ GeV}$, which means $3m_q + m_J \simeq 1 \text{ GeV}$ in the SJM. First we prove the property, assumed in the previous sections, that $\delta \equiv 2m_q - m_{IJ}$ used in Eqs. (2.4) and (2.5) is minor compared with $m_B \sim 1 \text{ GeV}$, which holds in a way irrelevant to the total length of the strings L_{min} . Next we discuss its implications and make the estimate of respective parameters by taking two typical values of L_{min} .

1. Substituting the relation $m_J = m_B - 3m_q$ into the expression of $\mathcal{E} = 2m_J + 3m_{IJ} \sim 2 \text{ GeV} \simeq 2m_B$, we have $2m_q \sim m_{IJ}$. Then we obtain the relation $\delta = 2m_q - m_{IJ} \sim 0$, namely, $\delta = O(0.1 \text{ GeV})$. This shows that $|\delta| \ll m_B$ holds, irrespective of L_{min} .
2. The relation $2m_q \sim m_{IJ}$ means that the energy for the fusion of q and \bar{q}

accompanying strings is almost equal to the energy of the inter-junction string m_{IJ} . Owing to this property, we can show that all the states in the \mathcal{M} series shown in Fig. 4 have the energy of about 2 GeV. Accordingly they can transform to each other without large energy transfer.

3. By taking two typical values of $L_{\min} \simeq 1.0$ fm and 1.5 fm, we give some estimate for the SJM parameters. $L_{\min} \simeq 1.0$ fm (1.5 fm) corresponds to the string length from the junction to the quark, $\ell_s \simeq L_{\min}/3 \simeq 0.33$ fm (0.5 fm) in the ground state of B . As noted already, the energy increase due to the string stretching is of the order of 1 GeV/fm. The string energy in B is $3m_q \simeq \sigma L_{\min} \simeq 1$ GeV (1.5 GeV), which leads to $m_J \sim 0$ (-0.5 GeV). The color Coulomb force gives the moderate attractive energy of about -0.2 GeV (-0.1 GeV).⁵⁾ The effect is much weaker than those (~ -0.5 GeV) obtained in calculations with use of the quark cluster model and the bag model,³⁶⁾ because the probability of close approach of the quarks is much smaller for the Y-shaped configuration. Energy contribution to m_J , being ~ 0.2 GeV (-0.4 GeV), is to be provided as energy concentration at the junction. Using the relation $2m_q \sim m_{IJ}$, we can estimate $m_{IJ} \sim 0.7$ GeV (1 GeV), the energy needed for formation of an interjunction.

Thus there remains a problem how to understand the value of m_J dependent on the choice of L_{\min} . As mentioned in §3, following the lattice QCD calculations,¹⁶⁾ the case of $L_{\min} \simeq 1.5$ fm seems favorable, since for $L_{\min} \simeq 1.0$ fm the string length ℓ_s is comparable with the width of the flux tube and the string-junction picture looks not so apparent. For the case of $L_{\min} \simeq 1.5$ fm, the negative energy of the order of $m_J \sim -0.4$ GeV is inevitable, since the string energy is considerably larger than m_B , but this feature is not unreasonable in the following sense. The negative m_J means the concentration of negative energy at the junction in the ground state of B without interjunction. In the exotic baryon states, however, the formation of new junctions inevitably accompanies the creation of interjunction(s) which needs the large positive energy in unit of $m_{IJ} \sim 1$ GeV.

Although the value of m_J looks directly linked to the parameter C_{3q} in Eq. (2·2), estimate of m_J from C_{3q} is not possible because C_{3q} contains an integration constant. Various effects to the energy of the ground state of B not specified in the simple mass formula in the SJM would contribute to m_J , and this problem is beyond the scope of the present study. As noted already this problematic point is not obstacle in the present work, because we have used the quantity \mathcal{E} preferentially.

6.2. On the BB repulsive core

The NN repulsive core is one of the main factors leading to the property that the nucleus with the low density is the lowest state in hadronic matter.^{*)} On the BB repulsive core, there have been rarely studied by taking into account the confinement mechanism, which is the nonperturbative effect originating from the color degrees of freedom in QCD. In the studies in the quark cluster model, although the quark

*) In the 1980's there was pointed out the possibility that the strange quark matter with almost equal numbers of u , d and s is the absolutely lowest state of hadronic matter. In a previous study with use of the quark cluster model and the bag model,³⁶⁾ we have shown that this possibility is not realized.

confinement is incorporated in a potential form between quarks, the interaction between clusters (baryons) is described in such a way that the confinement plays no substantial role as far as the color singlet is imposed to the total system, in spite of its significant contribution to the internal energy of the cluster of quarks (baryon). Since these studies have been developed to a stage that most of the existing data of the BB system are well explained,^{37),38)} the important findings are surely contained in the results of these works. Nevertheless it seems strange that the effects of the confinement does not appear in any essential manner in the BB interaction. We feel that something essential may be lacked in the recognition hitherto obtained. In the present paper we have shown a straightforward way to understand the BB repulsive core as originating from the confinement.

Recently the first realistic approach on nuclear force in lattice QCD calculations has been made in the quenched approximation.³⁹⁾ Although the outside attraction and the inside repulsion of the NN S -wave interaction have been reported, it is premature to discuss the results in quantitative level, and we expect future progress. From the viewpoint of the present paper, it is desirable to see at what extent the process like $J\bar{J}$ -pair creation takes part on the short-range interaction, although it will be a task beyond the quenched approximation. This problem is connected with the study to clarify whether exotic dibaryons exist or not. It seems quite natural that the lattice QCD simulation has not been performed yet concerning the exotic dibaryons, since they appear through the virtual creation of the $B\bar{B}$ pair and that of the $q\bar{q}$ pairs, following the context of the present paper. To get observational evidence on these exotic dibaryon states, we expect future experiments.^{*)} Concerning the dibaryon problems, it seems promising to obtain exclusive data with use of the anti-proton beam such as $\bar{p} + {}^3\text{He} \rightarrow D_4^6 + \gamma$ and $\bar{p} + {}^4\text{He} \rightarrow D_4^6 + N$.

6.3. On three-baryon interaction

Hitherto in most cases, the three-body interaction in the BBB system has been introduced as a supplementary effect to reproduce the data in a quantitative level. In recent years, its indispensable role in the nuclear system has been recognized progressively, concerning the saturation problem, the maximum mass of neutron stars, and the requirement to reproduce various kinds of precise measurements on few-nucleon systems. Nevertheless, it seems fair to ask why three-baryon interaction plays a decisive role in the fundamental aspects of the nucleus. We can accept that its importance is natural, if its short-range part originates from the confinement and its intermediate part comes from meson exchange, especially pion exchange.

6.4. On exotic multi-baryon system

In this paper we have treated the exotic baryon systems of the baryon number $N = 2, 3$ in the SJM. What aspects of such exotic systems come about when N becomes much larger? We can apply the same procedure as shown in Fig. 6 to

^{*)} Recently Imachi, Otsuki and Toyoda have studied unconventional hadrons including exotic ones, extending the string-junction model they developed in the 1970's.⁴⁰⁾ Further study by assigning the recently observed peak at the mass 4.43 GeV (Z^\pm)⁴¹⁾ as the exotic meson M_4^2 (e.g., $udc\bar{c}$ for Z^+), has suggested the appearance of the dibaryon D_6^4 here discussed at the mass about 4.3 GeV.⁴²⁾

the $N = 4$ system, and find that its energy is higher than the ordinary value $4m_B$ by $\sim 3\mathcal{E}$. Furthermore we can extend the treatment to the general case. Such an exotic multi-baryon system (abbreviated to EMBS), whose possibility was pointed out in Ref. 7c), have the energy higher than Nm_B of the ordinary baryon system by $(N - 1)\mathcal{E}$. Namely, EMBS has the saturation property of energy; $\mathcal{E}/\text{baryon} \sim 2$ GeV/baryon. However to estimate the saturation density of EMBS is problematic, contrary to the uniform matter. The reason is that EMBS connected by strings, junctions and interjunction strings is flexible in spatial form like the “chain”. Therefore it seems difficult to consider EMBS beyond few-baryon systems, unless its spatial configuration is well defined.

§7. Concluding remarks

In this study we have shown a way to unifiedly understand the universal repulsive core of the BB interaction and the universal repulsion of the three-baryon interaction, standing on the string-junction model where the confinement mechanism in QCD is built in.

The key points of recognition are summarized as follows. In order for the confinement mechanism to persist at closest approach of baryons, the $B\bar{B}$ -pair (more widely the pair of the junction and anti-junction) excitation takes place and leads to the formation of a new closed string-junction net with exotic nature. For the BB system the energy needed for the $B\bar{B}$ -pair creation appears as the repulsive core with ~ 2 GeV height. For the BBB system a half of the energy for the $2B\bar{B}$ -pair, being ~ 2 GeV, is regarded as the strength of the three-body potential. Participation of such exotic baryonic states brings about the latent effect on the ordinary baryon states, which appears as the short-range repulsion. Thus we can understand the universal (flavor- and spin- independent) short-range repulsion as originating from the confinement mechanism. Results of numerical calculations show that the universal repulsion of three-baryon interaction derived in the string-junction model provides us with the sufficiently stiff equation of state, which is consistent with the observations of neutron star masses, even when the hyperon-mixing occurs in the neutron star core.

We have developed our studies on the assumption of the existence of exotic multi-baryon states. Hitherto experimental evidence about exotic hadrons has not been obtained yet. Since there is no reason in QCD to deny the existence of hadrons other than the ordinary ones, this is a basic problem to be solved in future in hadron physics. Recently exotic hadrons have attracted much interest, experimentally and theoretically,⁴³⁾ and we hope that active studies now being in progress will elucidate such basic problems in future.

Concerning the problem under investigation, the reasoning taken here will not lose validity even though exotic hadron states with narrow width are not observed. The reason is that, e.g. for the BB case, if there exist a group of states with a junction number different from that of the ordinary BB case almost restricted to the small spatial space corresponding the core region of BB and their energies are as high as 2 GeV, the ordinary BB system can scarcely enter into such region. This

implies the existence of the repulsive core.

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